

Some Considerations on Chiral Gauge Theories

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Abstract

Some general considerations on the problem of non-perturbative definition of Chiral Gauge Theories are presented.

1 Introduction

While, classically, Vector and Chiral Gauge Theories look quite similar, their quantization proceeds through quite different lines, due to the phenomenon of chiral anomalies, which is reflected in the lack of a chiral invariant regularization.

A non-perturbative formulation of Chiral Gauge Theories could clarify fundamental issues, as the possibility of dynamical Higgs mechanism, baryon non conservation and the question of naturalness.

How should we quantize chiral gauge theories?

Several approaches have been explored:

1. Non gauge invariant quantization[1],[2],[3] (Rome approach¹) based on the Bogolubov method²
2. Gauge invariant quantization[6],[7]
3. Mirror Fermions[8]
4. Fine-Grained Fermions[9],[10],[11]
5. Other dimensions or infinite number of regulators[12],[13]
6. Overlap[14],[15],[16]
7. Ginsparg-Wilson Fermions[17],[18],[19]

The Overlap and the Ginsparg-Wilson approach make in fact use of the same fermion discretization, while differ in the treatment of the residual gauge invariance breaking, present at the lattice level.

In the following I will present some general considerations on the problem of quantization of Chiral Gauge Theories and on some of its solutions presented so far.

2 Target Theory

The first step, common to every approach, is to define the target theory we want to reproduce in the continuum. The target theory is usually identified through its symmetries and is defined by a formal, continuum, Lagrangian density:

$$L_g = \bar{\psi}_L \not{D} \psi_L + \frac{1}{4} W_{\mu\nu}^a W_{\mu\nu}^a \quad (1)$$

where:

$$D_\mu \equiv \partial_\mu + i g_0 W_\mu^a T^a \quad (2)$$

In eq.(2), the T^a 's are the appropriate generators of the gauge group G and g_0 denotes the bare coupling. Higgs degrees of freedom could easily be added to the action in eq.(1), but for simplicity we will not do so.

¹Within this class falls also the formulation of the Zaragoza group[4].

²Recent work in the framework of the Rome approach, including numerical simulations, has been carried on by the authors of ref.[5].

The general problem of the quantization of chiral theories is well known: regularization is, in general, incompatible with exact chiral invariance. This incompatibility is unavoidable and is at the origin of anomalies.

For example, naive discretization of Dirac action suffers from the so-called Doubling Problem: the spectrum of naively discretized fermions is vector-like rather than chiral.

A possible solution to this problem consists in the introduction of the so-called Wilson term[20]:

$$L_W \approx r\bar{\psi}_L a \partial^2 \psi_R + h.c. \quad (3)$$

or, in the Wilson Majorana form[21],

$$L_W = a \left(\chi^\alpha \partial_\mu \partial_\mu \chi_\alpha + \bar{\chi}^{\dot{\alpha}} \partial_\mu \partial_\mu \bar{\chi}_{\dot{\alpha}} \right) \quad (4)$$

where $\chi^\alpha(x)$ and $\bar{\chi}^{\dot{\alpha}}(x)$ are bispinor Grassmann fields. L_W is a chiral violating[22], "irrelevant" term: its presence can, and must, be compensated by finite or power divergent counterterms. No new logarithms appear as will be discussed later on. The Overlap[14],[15] and the Ginsparg-Wilson[18] formulation are based on a different discretization which captures more geometrical meaning with respect to the Dirac-Wilson one, but is confronted with a similar problem: the unmodified action S of an anomaly free chiral gauge theory has the form:

$$S = S_{GI} + a \int d^4x O_5(x) \quad (5)$$

where $O_5(x)$ is a non gauge invariant operator of dimension 5. As it will be discussed in the following sections, the presence of an uncompensated aO_5 term is, potentially, very dangerous.

3 Uncompensated Symmetry Breaking

Let us consider, e.g., a $\lambda\phi^4$ theory symmetric under $\phi \leftrightarrow -\phi$. As an example of what happens if the symmetry breaking induced by the regulator is not compensated by the introduction of appropriate counterterms, we may study the consequences of adding to the Lagrangian density an irrelevant term $aO_5 \propto a\phi^5$, which violates this symmetry.

In order to be really a correction of order a , O_5 should be multiplicatively renormalizable in the continuum limit ($a \rightarrow 0$), in order to avoid an immediate back-reaction, through mixing, giving rise to terms $O_3 \approx \phi^3$ and $O_1 \approx \phi$.

It will consist, therefore, of an appropriate linear combination of ϕ^5 , ϕ^3 and ϕ .

We will study the resulting theory by expanding Green's functions in powers of $a \int dx O_5$, while keeping the dependence on the coupling constant completely non-perturbative. Let us consider, in particular, the three-point Green's function which is expected to vanish in the symmetric $\phi \leftrightarrow -\phi$ theory:

$$\langle \phi(x_1)\phi(x_2)\phi(x_3) \rangle = \sum_{n=0}^{\infty} \frac{1}{n!} \left\langle \phi(x_1)\phi(x_2)\phi(x_3) (a \int dy O_5(y))^n \right\rangle \quad (6)$$

We will now discuss what happens at various orders in the insertion of $a \int dx O_5$.

- First order correction

$$\langle \phi(x_1)\phi(x_2)\phi(x_3) \rangle_{(1)} = a \int dy \langle \phi(x_1)\phi(x_2)\phi(x_3) O_5(y) \rangle \quad (7)$$

This is O.K. (it vanishes as $a \rightarrow 0$) since O_5 is multiplicatively renormalizable and, therefore, its single insertion is only logarithmically divergent.

- Higher order corrections

The next interesting contribution, in this particular example, is the third order insertion:

$$\langle \phi(x_1)\phi(x_2)\phi(x_3) \rangle_{(3)} = a^3 \int dy_1 dy_2 dy_3 \langle \phi(x_1)\phi(x_2)\phi(x_3) O_5(y_1) O_5(y_2) O_5(y_3) \rangle \quad (8)$$

The r.h.s of eq.(8), although formally of order a^3 , is in fact divergent and violates the symmetry $\phi \leftrightarrow -\phi$.

This follows from the fact, in eq.(8), the contribution from the integrals over y_1, y_2, y_3 , when the y 's are close together, is enhanced by short distance operator singularities, and can be estimated through power-counting³:

$$\int dy_1 dy_2 dy_3 O_5(y_1) O_5(y_2) O_5(y_3) \underset{a \rightarrow 0}{\approx} \frac{c}{a^4} \int dy O_3(y) \quad (9)$$

with an appropriately chosen c .

Using the Renormalization Group we will now sketch the proof that c can only depend on g_0 [23]. In fact, if we denote by Z_5 the logarithmically

³Eq.(9) is meant as an illustration only. In the r.h.s. there will also be a contribution from O_5 , which will not survive in eq.(14) and a contribution from O_1 , even more singular, which obviously leads to the same qualitative conclusions.

divergent renormalization constant which makes the single insertion of O_5 finite, we have that the triple composite operator insertion:

$$I(y_1, y_2, y_3) \equiv Z_5^3 O_5(y_1) O_5(y_2) O_5(y_3) \quad (10)$$

is finite (as $a \rightarrow 0$), but not integrable, i.e. it is not a distribution and $\int dy_1 dy_2 dy_3 I(y_1, y_2, y_3)$ does not exist in the continuum. $I(y_1, y_2, y_3)$ may, however, be transformed into a distribution through an appropriate subtraction such that:

$$T \equiv Z_5^3 \left(\int dy_1 dy_2 dy_3 O_5(y_1) O_5(y_2) O_5(y_3) - \frac{c}{a^4} \int dy O_3(y) \right) \quad (11)$$

stays finite when $a \rightarrow 0$, when inserted in a particular Green function.

c is dimensionless and may, a priori, depend on g_0 and $a\mu$:

$$c = c(g_0, a\mu) \quad (12)$$

We then have:

$$\int dy_1 dy_2 dy_3 O_5(y_1) O_5(y_2) O_5(y_3) = \frac{1}{Z_5^3} T + \frac{c}{a^4} \int dy O_3(y) \quad (13)$$

so that:

$$\begin{aligned} a^3 \int dy_1 dy_2 dy_3 O_5(y_1) O_5(y_2) O_5(y_3) &= \frac{a^3}{Z_5^3} T + \frac{c}{a} \int dy O_3(y) \approx \\ &\underset{a \rightarrow 0}{\approx} \frac{c}{a} \int dy O_3(y) \end{aligned} \quad (14)$$

Let us now consider the Callan-Symanzik differential operator[24]:

$$\mu \frac{d}{d\mu} \Big|_{g_0, a} \equiv \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \quad (15)$$

As evident from eq.(15), $\mu \frac{d}{d\mu} \Big|_{g_0, a}$ transforms u.v. finite quantities into finite quantities. Applying it to T , eq.(11), we get⁴:

$$\begin{aligned} \mu \frac{dT}{d\mu} \Big|_{g_0, a} &= 3\mu \frac{d \log Z_5}{d\mu} \Big|_{g_0, a} T - Z_5^3 \frac{\int dy_1 O_3(y_1)}{a^4} \mu \frac{dc}{d\mu} \Big|_{g_0, a} = \\ &= 3\gamma_{O_3} T - Z_5^3 \frac{\int dy_1 O_3(y_1)}{a^4} \mu \frac{dc}{d\mu} \Big|_{g_0, a} \end{aligned} \quad (16)$$

⁴Eq.(16) must be considered as a symbolical equation. The complete argument would require the insertion of T into a Green function. For a thorough treatment see ref.[23].

In eq.(16), when $\mu \frac{d}{d\mu} \Big|_{g_0, a}$ acts on bare, unrenormalized quantities like $O_5(y_1)O_5(y_2)O_5(y_3)$ and $\int dy O_3(y)$, gives 0 because they only depend from a and g_0 .

Eq.(16) shows that $\mu \frac{dT}{d\mu} \Big|_{g_0, a}$ is u.v. finite only if:

$$\mu \frac{dc}{d\mu} \Big|_{g_0, a} = 0 \quad (17)$$

i.e., if c is a function of g_0 only:

$$c = c(g_0) \quad (18)$$

The integration region where the y 's are close together, which is the only one surviving the continuum limit in eq.(8), gives therefore rise to a linearly divergent contribution:

$$\langle \phi(x_1)\phi(x_2)\phi(x_3) \rangle_{(3)} \approx \frac{c(g_0)}{a} \int dy \langle \phi(x_1)\phi(x_2)\phi(x_3)O_3(y) \rangle \quad (19)$$

with a coefficient depending on the bare parameters only.

Depending on the particular regularization employed, the appearance of these unwanted contributions can be shifted to higher orders in perturbation theory, but can hardly be eliminated, unless some exact selection rule is operating at the level of the regularized theory.

In order to treat the similar problem arising in Chiral Gauge Theories, the following strategies have been proposed:

- Add to the action compensating counterterms⁵;
- Gauge average

In the first class are included, although with quite a different status, the Rome approach[1] and the Lüscher[18] approach, based on the Ginsparg-Wilson[17] discretization. The use of Ginsparg-Wilson fermions allows, as shown by Lüscher[18], the very interesting possibility of compensating the lack of gauge invariance through the addition of appropriate counterterms,

⁵In order to show in a completely non-perturbative way that the adjustment of the coefficients of local counterterms in the action allows the elimination of such unwanted contributions, one should show that the symmetry breaking insertions exponentiate. This can be done following a line of reasoning similar to the one illustrated in sect.5.

thus obtaining a gauge invariant theory at finite lattice spacing. Through this procedure it is possible to avoid the necessity of a gauge fixing term, which is peculiar of the Rome approach.

Neuberger proposal[15], instead, is based on the conjecture that the breaking of gauge invariance takes care of itself, through averaging, according to a mechanism first suggested in ref.[25].

4 Rome Approach

In this section I will recall the basic strategy followed within the Rome approach[1].

One starts defining the vector fields through the link variables as e.g.:

$$ag_0 W_\mu \equiv \frac{U_\mu - U_\mu^+}{2i} \quad (20)$$

and then adds to the action the discretization of the continuum gauge-fixing term:

$$L_{gf} = \frac{1}{2\alpha_0} (\partial_\mu W_\mu^a)(\partial_\mu W_\mu^a) + \bar{c} \partial_\mu D_\mu c \quad (21)$$

The gauge-fixing Lagrangian in eq.(21) can be linearized through a set of Lagrange multipliers $\lambda^a(x)$, as:

$$L_{gf} = \frac{\alpha_0}{2} \lambda^a \lambda^a + i \lambda^a (\partial_\mu W_\mu^a) + \bar{c} \partial_\mu D_\mu c \quad (22)$$

Beside the term written in eq.(22), one also adds all possible symmetry breaking (and non-Lorentz invariant in the case of the Lattice Discretization) counterterms with dimension $D \leq 4$. As examples we mention:

$$- \frac{\delta \mu_W^2}{2} W_\mu^a(x) W_\mu^a(x) \quad (23)$$

$$\delta Z_1 \left(\partial_\mu W_\mu^a \right)^2 \quad (24)$$

$$\delta Z_2 f^{abc} \partial_\mu W_\nu^a W_\mu^b W_\nu^c \quad (25)$$

$$\delta Z_3 \sum_\mu \partial_\mu W_\mu^a \partial_\mu W_\mu^a \quad (26)$$

$$\delta g_{gh} f_{abc} \bar{c}^a \partial_\mu \left(W_\mu^b c^c \right) \quad (27)$$

The presence of the counterterm with δg_{gh} , eq.(27), signals an irreversible breaking of gauge geometry, within this approach, as discussed later.

The target gauge fixed theory is invariant under the BRST symmetry[26]:

$$\begin{aligned}
\delta\chi^\alpha &= \varepsilon\delta_{BRST}\chi^\alpha = i\varepsilon g_0 c^a T^a \chi^\alpha \\
\delta W_\mu^a &\equiv \varepsilon\delta_{BRST}W_\mu^a = \varepsilon(D_\mu c)^a \\
\delta\lambda^a &= 0 \\
\delta c^a &\equiv \varepsilon\delta_{BRST}c^a = -\frac{1}{2}\varepsilon g_0 f_{abc}c^b c^c \\
\delta\bar{c}^a &\equiv \varepsilon\delta_{BRST}\bar{c}^a = \varepsilon i\lambda^a
\end{aligned} \tag{28}$$

As well known these transformations are nilpotent:

$$\delta_{BRST}^2 = 0 \tag{29}$$

so that the target gauge fixing action, eq.(22), which can be rewritten as

$$L_{gf} = \frac{\alpha_0}{2}\lambda^a\lambda^a + \delta_{BRST}(\bar{c}^a\partial_\mu W_\mu^a) \tag{30}$$

is BRST-invariant:

$$\delta_{BRST}L_{gf} = 0 \tag{31}$$

BRST invariance implies, in the target theory, an infinite set of identities on the Green Functions:

$$\langle\Phi_1(x_1)\dots\dots\Phi_n(x_n)\rangle \equiv \int d\mu e^{-S_{cl}}\Phi_1(x_1)\dots\dots\Phi_n(x_n) \tag{32}$$

as:

$$\langle\delta_{BRST}(\Phi_1(x_1)\dots\dots\Phi_n(x_n))\rangle = 0 \tag{33}$$

The strategy, then, is to fix the values of the counterterms as follows. We compute, first of all, non-perturbatively:

$$\begin{aligned}
&\langle\Phi_1(x_1)\dots\dots\Phi_n(x_n)\rangle = \\
&= \int DU_\mu D\bar{\chi} D\chi D\bar{c} Dc \\
&e^{-S_0+S_W+\frac{1}{2\alpha_0}\int d^4x(\partial_\mu W_\mu^a)^2+S_{ghost}+S_{c.t.}}\Phi_1(x_1)\dots\dots\Phi_n(x_n)
\end{aligned} \tag{34}$$

and tune the values of the counterterms by imposing the BRST-”Identities”, eq.(33). This is at best possible up to order a , and impossible if there are unmatched anomalies.

This procedure defines a bare chiral theory with parameters g_0 and α_0 defined by the BRST transformations. We can then carry out the usual non-perturbative renormalization, by fixing the independent bare parameters g_0 and α_0 to reproduce given finiteness conditions on physical quantities and/or Green functions.

I want to stress that this procedure is completely non-perturbative, but can also be checked in Perturbation Theory[1],[27]. Due to the rather large number of possible counterterms, the perturbative evaluation of some of them might be helpful in practice. In fact the theory so defined should be asymptotically free and we may distinguish two different kinds of counterterms:

- Dimensionless counterterms:

$$\delta Z = f_Z(g_0, \alpha_0) \quad (35)$$

like those appearing in eqs.(24)-(27).

- Dimensionful counterterms:

$$\delta M = \frac{f_M(g_0, \alpha_0)}{a} \quad (36)$$

like the one in eq.(23). While dimensionless counterterms can be safely estimated in perturbation theory, since $g_0 \rightarrow 0$, the dimensionful counterterms are essentially non-perturbative. In fact exponentially small contributions to f_M can be rescued by $\frac{1}{a}$:

$$\frac{f(g_0, \alpha_0)}{a} \approx \frac{e^{-\frac{1}{g_0^2}}}{a} \approx \Lambda \quad (37)$$

where Λ is a non-perturbative parameter of the same nature as Λ_{QCD} .

To conclude this section, I would like to add that ghosts and gauge-fixing are unavoidable features of the Rome approach. In fact in a two loop computation in dimensional regularization[28] the ghost counterterm, eq.(27), has been shown to be present, so that, at least if we trust Perturbation Theory, we cannot invert the Faddeev-Popov procedure and remove the gauge fixing.

5 Gauge-Average without Gauge-Fixing

Another possible way to cope with $O(a)$ violations of chiral gauge invariance due to the regulator, consists in the use of the gauge non-invariant

theory[25],[6],[7],[15]:

$$S = S_{GI} + a \int d^4y O_5(y) \quad (38)$$

without any gauge-fixing⁶. According to the Nielsen-Ninomiya theorem[22], the theory described by S_{GI} alone would be vector-like because of the presence of doublers.

The basic idea behind the strategy of gauge-average is the possibility that the gauge invariant integration measure in the functional integral, will automatically take care of the "small" violations induced by the regulator, leading, in the end, to the correct theory. This mechanism is essentially non-perturbative and, therefore, it stands like a conjecture, difficult to prove and difficult to disprove. In the following I will present some heuristic remarks about this conjecture.

For an observable, gauge invariant quantity, O_{GI} , the procedure of using eq.(38) without any gauge-fixing, is in fact equivalent to an average over gauge transformations Ω :

$$\begin{aligned} \langle O_{GI} \rangle &\equiv \int DU D\bar{\chi} D\chi e^{S_{GI} + a \int dy O_5(y)} O_{GI} = \\ &= \int DU D\bar{\chi} D\chi D\Omega e^{S_{GI} + a \int dy O_5(y)^\Omega} O_{GI} \end{aligned} \quad (39)$$

Since the theory is lattice regulated, the Ω -integration in eqs.(39) is compact and obeys the rules of group theory, as, e.g.:

$$\int D\Omega \Omega_{ij}(x) \Omega_{kl}^\dagger(y) = \delta_{xy} \delta_{il} \delta_{jk} \quad (40)$$

We have now to study the general form of the insertion $\int D\Omega e^{a \int dy O_5(y)^\Omega}$ in eq.(39). To do this we expand the integrand in powers of $a \int dy O_5(y)^\Omega$:

$$\langle O_{GI} \rangle = \sum_{n=0}^{\infty} \langle O_{GI} \rangle_{(n)} = \sum_{n=0}^{\infty} \frac{a^n}{n!} \int D\Omega \left\langle O_{GI} \left(\int d^4y O_5(y)^\Omega \right)^n \right\rangle \quad (41)$$

The Ω integration in eq.(41) projects the gauge invariant contribution out of $O_5(y)$ and their products. In virtue of eq.(40) a string of $O_5(y)$'s will, after gauge average, cluster into a sum of local gauge invariant operators multiplied by space-time δ -functions.

⁶We are using continuum notations for the lattice regulated theory.

As an example, let us first consider a simplified situation in which $\int D\Omega O_5(x)^\Omega = 0$ and $\int D\Omega O_5(x)^\Omega O_5(y)^\Omega = F(x)\delta(x-y)$ with a local gauge invariant $F(x)$. In this case it is easy to resum the series in eq.(41):

$$\begin{aligned}
\int D\Omega e^{a \int dy O_5(y)^\Omega} &= \frac{1}{2!} a^2 \int dx_1 dx_2 \int D\Omega O_5(x_1)^\Omega O_5(x_2)^\Omega + \\
&+ \frac{1}{4!} a^4 \int dx_1 \dots dx_4 \int D\Omega O_5(x_1)^\Omega \dots O_5(x_4)^\Omega + \dots = \\
&= \int dx_1 a^2 \frac{F(x_1)}{2} + \frac{1}{2!} \int dx_1 dx_2 a^2 \frac{F(x_1)}{2} a^2 \frac{F(x_2)}{2} + \dots = \\
&= e^{\int dy a^2 \frac{F(y)}{2}} \quad (42)
\end{aligned}$$

In the general case the structure displayed in eq.(42) is maintained, with $a^2 \frac{F(y)}{2}$ replaced by an infinite series of irrelevant, local, gauge invariant operators obtained by projecting the gauge invariant contribution out of the product of an increasing number of O_5 's. Although local, these operators have an increasing spread in y and the question is if they can really be treated as irrelevant insertions. If they could, then, as far as the expectation value of gauge invariant observables is concerned, the theory would be in the same universality class as the one with the exactly gauge invariant action S_{GI} in eq.(38). Therefore the doublers would be back and we would end up with a Vector-like Spectrum. The answer to this question, as remarked before, lies outside the scope of perturbation theory. In particular there could exist different fixed points, with different anomalous dimensions, corresponding to different continuum theories. Although conceivable, this possibility looks rather unlikely when, as in the present case, the theory to be defined is asymptotically free and therefore controlled by the fixed point at zero coupling.

6 Conclusions

The quantization of Chiral Gauge Theories is still a fascinating and evolving subject. We are witnessing very interesting progress, with possible applications also in the framework of consolidated vector theories as, for instance, QCD.

It must be remarked, however, that even these new developments do not put Chiral and Vector theories on the same ground and seem to maintain

some basic, deep difference, in particular in what concerns the subjects of fine tuning and naturalness.

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References

- [1] A. Borrelli, L. Maiani, G.C. Rossi, R. Sisto, M. Testa, *Nucl. Phys. B* **333**, 335 (1990).
- [2] A. Borrelli, L. Maiani, G.C. Rossi, R. Sisto, M. Testa, *Phys. Lett. B* **221**, 360 (1989).
- [3] M. Testa, *The Rome Approach to Chirality*. In *Seoul 1997, Recent developments in nonperturbative quantum field theory* 114, hep-lat/9707007
- [4] J.L. Alonso, P. Boucaud, J.L. Cortes, E. Rivas, *Nucl. Phys. B* (Proc. Suppl.) **17**, 461 (1990).
- [5] W. Bock, M. F. L. Golterman, Y. Shamir, *Phys. Rev. Lett.* **80**, 3444 (1998).
- [6] J. Smit, Seillac Conf. 1987; *Nucl. Phys. B* (Proc. Suppl.) **4**, 451 (1988); P.D.V. Swift, *Phys. Lett. B* **145**, 256 (1984).
- [7] S. Aoki, I.H. Lee, S.S. Xue, *Phys. Lett. B* **229**, 77 (1989).
- [8] I. Montvay, *Phys. Lett. B* **199**, 89 (1987); *Phys. Lett. B* **205**, 315 (1988).
- [9] M. Gockeler, G. Schierholz, in *Nonperturbative Aspects of Chiral Gauge Theories, Rome, Italy, Mar 9-11, 1992* *Nucl. Phys. B* (Proc. Suppl.) **29B/C** (1992).
- [10] G. 't Hooft, *Phys. Lett. B* **349**, 491 (1995).
- [11] P. Hernandez, R. Sundrum, *Nucl. Phys. B* **455**, 287 (1995).

- [12] D.B. Kaplan, *Phys. Lett. B* **2**, 8 (8) 342 (1992).
- [13] S.A. Frolov, A.A. Slavnov, *Nucl. Phys. B* **411**, 647 (1994).
- [14] R. Narayanan, H. Neuberger, *Phys. Rev. Lett.* **71**, 3251 (1993).
- [15] H. Neuberger, *Phys. Rev. D* **59**, 085006 (1999).
- [16] S. Randjbar-Daemi, J. Strathdee, *Nucl. Phys. B* **443**, 386 (1995).
- [17] P. H. Ginsparg, K. G. Wilson, *Phys. Rev. D* **25**, 2649 (1982).
- [18] M. Luscher, *Nucl. Phys. B* **549**, 295 (1999).
- [19] P. Hasenfratz, *Nucl. Phys. B* **525**, 409 (1998).
- [20] K. Wilson, *Phys. Rev. D* **14**, 2455 (1974); in *New phenomena in subnuclear physics*, ed. A. Zichichi (Plenum, New York, 1977).
- [21] L. Maiani, G.C. Rossi, M. Testa, *Phys. Lett. B* **292**, 397 (1992).
- [22] H.B. Nielsen, M. Ninomiya, *Nucl. Phys. B* **185**, 20 (1981); *Nucl. Phys. B* **195**, 541 (1982); *Nucl. Phys. B* **193**, 173 (1981).
- [23] M. Testa, JHEP 9804:002 (1998), hep-th/9803147.
- [24] C. G. Callan, *Phys. Rev. D* **2**, 1541 (1970) ;
K. Symanzik, *Comm. Math. Phys.* **18** (1970) 227.
- [25] D. Foerster, H.B. Nielsen, M. Ninomiya, *Phys. Lett. B* **94**, 135 (1980).
- [26] C. Becchi, A. Rouet, R. Stora, *Phys. Lett. B* **52**, 344 (1974);
I.V. Tyupkin, *Gauge invariance in field theory and statistical physics in operatorial formulation*, preprint of Lebedev Physics Institute n.39 (1975).
- [27] G. Travaglini, *Nucl. Phys. B* **507**, 709 (1997).
- [28] G. C. Rossi, R. Sarno, R. Sisto, *Nucl. Phys. B* **398**, 101 (1993).